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Hawking Radiation and Evaporation of the Black Hole Induced by a Klein-Gordon Soliton

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ABSTRACT

A two-dimensional dilatonic black hole induced by a topological soliton is exactly solvable in the scalar field theory coupled to dilaton gravity. The Hawking radiation of the black hole is studied in the one-loop approximation with the help of the trace anomaly of energy-momentum tensors which is a geometrical invariant. The quantum theory can be also soluble in the RST scheme in order to consider the back reaction of the metric. The energy of the black hole system is calculated and the classical thunderpop energy corresponding to the soliton energy is needed to describe the final state of the black hole. Finally we discuss the possibility of conservation of the topological charge.

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The dilaton gravity model coupled to the conformal fields given by Callan-Giddings-Harvey-Strominger (CGHS) [1] is very intriguing since this model possesses most of the interesting properties of the four-dimensional gravity theories such as a dynamical black hole solution and Hawking radiations [2] even though the system has fewer degrees of freedom compared to the four-dimensional gravity. The infalling matter fields of a shock wave form a dynamical black hole, and the back reaction of the metric due to the Hawking radiation in the one-loop approximation is described by the conformal anomaly term in the semiclassical action [1]. Russo-Susskind-Thorlacius (RST) [3] obtained an exact dynamical black hole solution at the quantum level by adding a local counter term to the CGHS model, and the back reaction of the metric and the final state of black hole have been well appreciated.

Recently, a two-dimensional dilaton gravity coupled to the solitons [4] was proposed as a generalization of the infalling shock wave and exactly solvable models. The massive infalling soliton induces the classical black hole solution. The quantum aspects of the black hole induced by the soliton especially for the case of Sine-Gordon model were studied by considering the Hawking radiation on the black-hole background in Ref. [5].

In this paper, we introduce a time-dependent classical black hole solution induced by the Klein-Gordon soliton having a lumped energy, which satisfies asymptotic flatness in our spacetime. Then the quantum-mechanical black hole solution incorporated with the back reaction of the metric due to the Hawking radiation is semiclassically obtained in the one-loop approximation. To obtain the exact solution, we use the fact that the trace anomaly of the matter fields is geometrically invariant quantity and the effective action may have a local counter term. The Hawking radiation and the Bondi energy reflecting the back reaction of the metric are calculated in the future null infinities and the final state of the black hole is discussed. Finally we debate the conservation of the topological soliton charge.

Let us now introduce the two-dimensional dilaton gravity coupled to the matter field,

$$\begin{aligned} S_{Cl} &= S_{DG} + S_f, \qquad (1) \\ S_{DG} &= \frac{1}{2\pi}\int d^2x\sqrt{-g}\left[e^{-2\phi}(R + 4(\bigtriangledown\phi)^2 + 4\lambda^2)\right], \end{aligned}$$

$$S_f = \frac{1}{2\pi}\int d^2x\sqrt{-g}\left[-\frac{1}{2}(\bigtriangledown f)^2 - e^{-2\phi}U(f)\right]$$

where g, ϕ, and f are metric, dilaton, and matter fields respectively. The classical equations of motion are given by

$$e^{-2\phi}[R_{\mu\nu} + 2\bigtriangledown_\mu \bigtriangledown_\nu \phi] = \frac{1}{2}\left(\bigtriangledown_\mu f \bigtriangledown_\nu f - \frac{1}{2}g_{\mu\nu}(\bigtriangledown f)^2\right), \tag{2}$$

$$R = 4\left[(\bigtriangledown\phi)^2 - \bigtriangledown^2\phi + \lambda^2\right] + U(f), \tag{3}$$

$$\bigtriangledown^2 f - e^{-2\phi}\frac{\delta U(f)}{\delta f} = 0. \tag{4}$$

In the conformal gauge, $g_{\pm\mp} = -\frac{1}{2}e^{2\rho}$ and $g_{\pm\pm} = 0$, the equations of motion then reduce to

$$T_{+-} = 2e^{-2\phi}\partial_+\partial_-(\rho - \phi) = 0, \tag{5}$$

$$T_{\pm\pm} = e^{-2\phi}\left(-2\partial_\pm^2\phi + 4\partial_\pm\rho\partial_\pm\phi\right) + \frac{1}{2}(\partial_\pm f)^2 = 0, \tag{6}$$

$$-4\partial_+\partial_-\phi + 4\partial_+\phi\partial_-\phi + 2\partial_+\partial_-\rho + \left(\lambda^2 - \frac{1}{4}U(f)\right)e^{2\rho} = 0, \tag{7}$$

$$\partial_+\partial_- f + \frac{1}{4}e^{2(\rho-\phi)}\frac{\delta U(f)}{\delta f} = 0. \tag{8}$$

In Eq. (5), we obtain the solution,

$$\rho = \phi + \frac{1}{2}\left(\omega_+(x^+) + \omega_-(x^-)\right) \tag{9}$$

where the holomorphic and antiholomorphic functions $\omega_\pm$ reflect the residual conformal degrees of freedom. In the Kruskal gauge $\omega_\pm = 0$, the equations of motion are most simply given by

$$\partial_\pm^2(e^{-2\phi}) + \frac{1}{2}(\partial_\pm f)^2 = 0, \tag{10}$$

$$\partial_+\partial_-(e^{-2\phi}) + \lambda^2 - \frac{1}{4}U(f) = 0, \tag{11}$$

$$\partial_+\partial_- f + \frac{1}{4}\frac{\delta U(f)}{\delta f} = 0. \tag{12}$$

Considering the potential $U(f) = \frac{\beta}{4}(f^2 - \frac{\mu^2}{\beta})^2$ in order to obtain the Klein-Gordon soliton solution [4] where the coupling constants μ and β are positive, the Klein-Gordon equation (12) yields an exact solution,

$$f(x^+, x^-) = \sqrt{\frac{\mu^2}{\beta}}tanh(\Delta - \Delta_0), \tag{13}$$

where $\Delta(x^+,x^-) = \frac{\mu}{2\sqrt{2}}(\alpha x^+ - \frac{1}{\alpha}x^-)$, $\alpha = \sqrt{\frac{1+v}{1-v}}$, $-1 < v < 1$, and $x_0^\pm$ implies the center of soliton coordinate. The range of infalling velocity of the soliton is $0 < v_{in} < 1$ and outgoing case is $-1 < v_{out} < 0$. Accordingly we denote Δ_{in} or Δ_{out} depending on the direction of velocity of soliton. From Eqs. (10) and (11), the infalling soliton soution (13) induces dilaton and metric solution, which is given by

$$\begin{aligned} e^{-2\phi(x^+,x^-)} &= e^{-2\rho(x^+,x^-)} \\ &= C + a_+x^+ + a_-x^- - \lambda^2 x^+x^- \\ &\quad -\frac{\mu^2}{12\beta}tanh^2(\Delta_{in}-\Delta_0) - \frac{\mu^2}{3\beta}lncosh(\Delta_{in}-\Delta_0) \end{aligned} \tag{14}$$

where C and $a_\pm$ are constants to be determined by boundary conditions.

For the limit $\Delta_{in} - \Delta_0 \ll -1$, we require the metric-dilaton solution to be an exact linear dilaton vacuum (LDV). By choosing the above constants as

$$C = \frac{\mu^2}{3\beta}(\Delta_0 + \frac{1}{4} - \ln 2), \quad a_+ = -\frac{\mu^3\alpha_{in}}{6\sqrt{2}\beta}, \quad a_- = +\frac{\mu^3}{6\sqrt{2}\alpha_{in}\beta}, \tag{15}$$

we obtain the LDV

$$e^{-2\phi} = -\lambda^2 x^+ x^-. \tag{16}$$

For the limit $\Delta_{in} - \Delta_0 \gg 1$, the definite black hole geometry appears as

$$e^{-2\phi} = \frac{M}{\lambda} - \lambda^2\left(x^+ - \frac{\mu^3}{3\sqrt{2}\alpha_{in}\beta\lambda^2}\right)\left(x^- + \frac{\mu^3\alpha_{in}}{3\sqrt{2}\beta\lambda^2}\right) \tag{17}$$

where $M = \frac{\mu^2\lambda}{3\beta}\left(2\Delta_0 - \frac{\mu^4}{6\beta\lambda^2}\right)$ is a ADM mass [6,17]. We assume that the black hole mass M is positive definite *i.e.*,

$$\alpha_{in}x_0^+ - \frac{1}{\alpha_{in}}x_0^- > \frac{\mu^3}{3\sqrt{2}\beta\lambda^2} \tag{18}$$

which is more explicitly, depending on the velocity of the infalling soliton, for $v_{in} \to 0$, $x_0^1 > \frac{\mu^3}{3\sqrt{2}\alpha_{in}\beta\lambda^2} > 0$ and for $v_{in} \to 1$, then $x_0^+ > \frac{\mu^3}{3\sqrt{2}\alpha_{in}\beta\lambda^2} > 0$. It means that the center of soliton $x_0^\pm$ can be located in our spacetime.

It seems to be appropriate to comment on the two limits. For $\Delta_{in} - \Delta_0 \ll -1$ equivalently $\alpha_{in}x^+ - \frac{1}{\alpha_{in}}x^- \ll \alpha_{in}x_0^+ - \frac{1}{\alpha_{in}}x_0^-$, this means that the region we consider is restricted to $x^1 \ll x_0^1$ for the small velocity of soliton or $x^+ \ll x_0^+$ for the speed of light. For $\Delta_{in} - \Delta_0 \gg 1$, the region is located in $x^1 \gg x_0^1$ for $v_{in} \to 0$ and $x^+ \gg x_0^+$ for $v_{in} \to 1$.

To study quantum effects of black hole, we follow Wald's axioms [5,7]. The two-dimensional trace of energy-momentum tensors is determined only by the curvature scalar R which is the only available geometric invariant,

$$T_\mu^\mu = \frac{1}{2}\kappa R \tag{19}$$

where κ is a constant which may be determined by some explicit calculation of trace anomaly. On the other hand, the ghost contribution to the trace anomaly (19) in the conformal gauge fixing can be canceled by adding a ghost decoupling term [8,9,14], $S_{St} = \frac{1}{\pi}\int d^2x\,[2\partial_+(\rho-\phi)\partial_-(\rho-\phi)]$. Anyway we assume that κ is positive to obtain a positive definite Hawking radiation.

We now consider the back reaction of the metric semiclassically by including the effect of trace anomaly (19) to the classical action. The quantum effective action is assumed to be

$$S_{Qt} = \frac{1}{2\pi}\int d^2x\sqrt{-g}\left[-\frac{\kappa}{4}R\frac{1}{\nabla^2}R - \frac{\gamma}{2}\phi R\right] \tag{20}$$

where we include a local counter term with a constant γ. In the conformal gauge, the total action reduces to

$$\begin{aligned} S_T &= S_{Cl} + S_{Qt} \\ &= \frac{1}{\pi}\int d^2x\left[e^{-2\phi}\left(2\partial_+\partial_-\rho - 4\partial_+\phi\partial_-\phi + (\lambda^2 - \frac{1}{4}U(f))e^{2\rho}\right)\right. \\ &+ \left.\frac{1}{2}\partial_+f\partial_-f - \kappa\partial_+\rho\partial_-\rho - \kappa\phi\partial_+\partial_-\rho\right] \end{aligned} \tag{21}$$

where we choose $\gamma = \kappa$ to solve exactly. Following the Bilal and Callan [10] and de Alwis's method [11], we perform field redefinition to a Liouville-like theory [3],

$$\begin{aligned} \Omega &= \frac{\sqrt{\kappa}}{2}\phi + \frac{e^{-2\phi}}{\sqrt{\kappa}}, \\ \chi &= \sqrt{\kappa}\rho - \frac{\sqrt{\kappa}}{2}\phi + \frac{e^{-2\phi}}{\sqrt{\kappa}}. \end{aligned} \tag{22}$$

Then, the action (21) and the two constraint equations (6) in terms of the redefined fields are given by

$$S_T = \frac{1}{\pi}\int d^2x\left[-\partial_+\chi\partial_-\chi + \partial_+\Omega\partial_-\Omega + (\lambda^2 - \frac{1}{4}U(f))e^{\frac{2}{\sqrt{\kappa}}(\chi-\Omega)} + \frac{1}{2}\partial_+f\partial_-f\right], \tag{23}$$

$$\kappa t_{\pm} = -\partial_{\pm}\chi\partial_{\pm}\chi + \partial_{\pm}\Omega\partial_{\pm}\Omega + \sqrt{\kappa}\partial_{\pm}^2\chi + \frac{1}{2}\partial_{\pm}f\partial_{\pm}f, \tag{24}$$

where $t_{\pm}$ reflect the nonlocality of the quantum effective action (20). From the action (23), we obtain equations of motion,

$$\begin{aligned}
\partial_+\partial_-\chi + \frac{1}{\sqrt{\kappa}}(\lambda^2 - \frac{1}{4}U(f))e^{\frac{2}{\sqrt{\kappa}}(\chi-\Omega)} &= 0, & (25)\\
\partial_+\partial_-\Omega + \frac{1}{\sqrt{\kappa}}(\lambda^2 - \frac{1}{4}U(f))e^{\frac{2}{\sqrt{\kappa}}(\chi-\Omega)} &= 0, & (26)\\
\partial_+\partial_- f + \frac{1}{4}e^{\frac{2}{\sqrt{\kappa}}(\chi-\Omega)}\frac{\delta U(f)}{\delta f} &= 0. & (27)
\end{aligned}$$

In the Kruskal gauge, we obtain the following solution,

$$\begin{aligned}
\Omega(x^+, x^-) &= \chi(x^+, x^-) \\
&= \frac{1}{\sqrt{\kappa}}\Big[C + a_+x^+ + a_-x^- - \lambda^2x^+x^- - \frac{\kappa}{4}\ln(-\lambda^2x^+x^-) \\
&\quad -\frac{\mu^2}{12\beta}\tanh^2(\Delta_{in}-\Delta_0) - \frac{\mu^2}{3\beta}lncosh(\Delta_{in}-\Delta_0)\Big] \qquad (28)
\end{aligned}$$

where the solution obeys the constraints (24) if $t_{\pm} = 0$.

To delineate the geometry given by (28), we consider asymptotic regions, in fact, the physically relevant quantities such as Hawking radiation and Bond energy [12] can be defined in these regions. In the asymptotic region, $\Delta_{in} - \Delta_0 \ll -1$, the solution Ω becomes the LDV,

$$\bar{\Omega} = -\frac{\lambda^2}{\sqrt{\kappa}}x^+x^- - \frac{\sqrt{\kappa}}{4}\ln(-\lambda^2x^+x^-). \tag{29}$$

On the other hand, for $\Delta_{in} - \Delta_0 \gg 1$, the definite black-hole geometry appears as

$$\Omega = \frac{M}{\sqrt{\kappa}\lambda} - \frac{\sqrt{\kappa}}{4}\ln(-\lambda^2x^+x^-) - \frac{\lambda^2}{\sqrt{\kappa}}\left(x^+ - \frac{\mu^3}{3\sqrt{2}\alpha_{in}\beta\lambda^2}\right)\left(x^- + \frac{\mu^3\alpha_{in}}{3\sqrt{2}\beta\lambda^2}\right), \tag{30}$$

where in the present limit, the region we consider is $x^1 \gg \frac{\mu^3}{3\sqrt{2}\alpha_{in}\beta\lambda^2}$ or $x^+ \gg \frac{\mu^3}{3\sqrt{2}\alpha_{in}\beta\lambda^2}$.

The singularity occurs at the boundary of the range of Ω where $\Omega_{min} = \frac{\sqrt{\kappa}}{4}(1-\ln\frac{\kappa}{4})$, which is given by

$$\begin{aligned}
\frac{\kappa}{4}(1-\ln\frac{\kappa}{4}) &= -\lambda^2\bar{x}^+(\bar{x}^- + \frac{\mu^3\alpha_{in}}{3\sqrt{2}\beta\lambda^2}) - \frac{\kappa}{4}\ln(-\lambda^2\bar{x}^+\bar{x}^-) \\
&+ \frac{\mu^3}{3\sqrt{2}\alpha_{in}\beta}\bar{x}^- + \frac{2\mu^2\Delta_0}{3\beta}. \qquad (31)
\end{aligned}$$

The location of the singularity is inside an apparent horizon which is given by another curve $\partial_+\Omega = 0$,

$$\lambda^2 \hat{x}^+ (\hat{x}^- + \frac{\mu^3 \alpha_{in}}{3\sqrt{2}\beta\lambda^2}) + \frac{\kappa}{4} = 0. \tag{32}$$

Following the suggestion of Hawking [13], RST showed that the singularity and apparent horizon collide in a finite proper time and the singularity is naked after the two have merged [3]. From (31) and (32), the following relation for the intersection point (x_s^+, x_s^-) is given

$$\frac{\kappa}{4} + \frac{\mu^3 \alpha_{in}}{3\sqrt{2}\beta} x_s^+ = \frac{\kappa}{4} e^{\frac{4M}{\kappa\lambda}} e^{-\frac{\mu^3 \kappa}{3\sqrt{2}\alpha_{in}\beta\lambda^2 x_s^+}} \tag{33}$$

and x_s^- coordinate is determined by Eq. (32).

In eq. (30), the first parenthesis of the third term is approximately written by

$$x^+ \left(1 - \frac{\mu^3}{3\sqrt{2}\alpha_{in}\beta\lambda^2 x^+} \right) \approx x^+ \tag{34}$$

if we mainly treat the high speed limit of the infalling soliton for convenience. Hereby, the geometry can be very similar to that of the RST model. We shall from now on consider the high speed case of $\Delta_{in} - \Delta_0 \gg 1$. Since we now consider the asymtotic region $\Delta_{in} - \Delta_0 \gg 1$ which is compatible with the range $x^+ \gg x_0^+ > \frac{\mu^3}{3\sqrt{2}\alpha_{in}\beta\lambda^2}$, the intersection point is approximately given by

$$\begin{aligned} x_s^+ &\approx \frac{3\sqrt{2}\beta\kappa}{4\mu^3 \alpha_{in}} (e^{\frac{4M}{\kappa\lambda}} - 1), \\ x_s^- &\approx -\frac{\mu^3 \alpha_{in}}{3\sqrt{2}\beta\lambda^2} \frac{1}{(1 - e^{-\frac{4M}{\kappa\lambda}})}. \end{aligned} \tag{35}$$

Note that Eq. (35) is valid only for

$$M \gg \frac{\kappa\lambda}{4} \ln \left(1 + \frac{2\mu^6}{9\beta^2\lambda^2\kappa} \right) \tag{36}$$

since x_s^+ belongs to the region $x^+ \gg \frac{\mu^3}{3\sqrt{2}\alpha_{in}\beta\lambda^2}$.

For the finial state of the black hole, a vacuum as a boundary condition is chosen in such a way that the continuity condition is satisfied between a shifted vacuum solution and the curved spacetime along some curve, where a shifted vacuum is chosen as

$$\begin{aligned} \bar{\Omega} &= -\frac{\lambda^2}{\sqrt{\kappa}} (x^+ - \frac{\mu^3}{3\sqrt{2}\alpha_{in}\beta\lambda^2})(x^- + \frac{\alpha_{in}\mu^3}{3\sqrt{2}\beta\lambda^2}) \\ &\quad -\frac{\sqrt{\kappa}}{4} \ln \left(-\lambda^2 (x^+ - \frac{\mu^3}{3\sqrt{2}\alpha_{in}\beta\lambda^2})(x^- + \frac{\alpha_{in}\mu^3}{3\sqrt{2}\beta\lambda^2}) \right). \end{aligned} \tag{37}$$

Along with the following curve, the geometry is continuous,

$$\frac{x^+x^-}{(x^+ - \frac{\mu^3}{3\sqrt{2}\alpha_{in}\beta\lambda^2})(x^- + \frac{\alpha_{in}\mu^3}{3\sqrt{2}\beta\lambda^2})} = e^{\frac{4M}{\kappa\lambda}} \tag{38}$$

where for $x^+ \gg \frac{\mu^3}{3\sqrt{2}\alpha_{in}\beta\lambda^2}$, the line is approximately straight line, $x^- \approx x_s^-$. Then the energy density of thunderpop is

$$\begin{aligned} \frac{1}{2}(\partial_- f)^2 &= \kappa t_- - \sqrt{\kappa}\partial_-^2\chi \\ &\approx -\frac{\kappa}{4}\frac{(1 - e^{-\frac{4M}{\kappa\lambda}})}{(x^- + \frac{\mu^3\alpha_{in}}{3\sqrt{2}\beta\lambda^2})}\delta(x^- - x_s^-). \end{aligned} \tag{39}$$

Before working out the Hawking radiation, we discuss coordinate transformations. The conformal transformations defined by $x^\pm = \pm\frac{1}{\lambda}e^{\pm\lambda\sigma^\pm}$ do not give an asymptotically static configuration and in particular the dilaton and graviton fields do not approach the correct form of LDV at infinity, so we introduce a quasi-static coordinate $y^\pm$ where the fields approach LDV in spatial and null infinities,

$$\begin{aligned} x^+ &= \frac{1}{\lambda}e^{\lambda y^+} + \frac{\mu^3}{3\sqrt{2}\alpha_{in}\beta\lambda^2}, \\ x^- &= -\frac{1}{\lambda}e^{-\lambda y^-} - \frac{\mu^3\alpha_{in}}{3\sqrt{2}\beta\lambda^2}. \end{aligned} \tag{40}$$

In this coordinate, Ω and χ are static to the leading order. We shall calculate the Hawking radiation and Bondi energy in the asymptotically static coordinate.

Let us now consider the Hawking radiation. From the fundamental condition that $T_{\pm\pm}$ must be true tensors without anomaly, we require the anomalous transformation as

$$t_\pm(y^\pm) = \left(\frac{\partial y^\pm}{\partial\sigma^\pm}\right)^{-2}\left(t_\pm(\sigma^\pm) - \frac{1}{2}D^s_{\sigma^\pm}(y^\pm)\right) \tag{41}$$

where the Schwarzian derivative is $D^s_{\sigma^\pm}(y^\pm) = \frac{y^{-'''}}{y^{-'}} - \frac{3}{2}(\frac{y^{-''}}{y^{-'}})^2$. Then, following [10], we obtain the Hawking radiation,

$$\begin{aligned} h(y^-) &= -\kappa t_-(y^-) \\ &= \frac{\kappa\lambda^2}{4}\left[1 - \frac{1}{(1 + \frac{\mu^3\alpha_{in}}{3\sqrt{2}\beta\lambda}e^{\lambda y^-})^2}\right] \end{aligned} \tag{42}$$

for $y^- < y_s^-$, and $h(y^-) = 0$ for $y^- > y_s^-$. As expected, for $y^- \to -\infty$, there is no Hawking radiation. For $y^- < y_s^-$, the integrated Hawking flux $H(y^-)$ is calculated as

$$\begin{aligned} H(y^-) &= \int_{-\infty}^{y^-} dy^- h(y^-) \\ &= \frac{\kappa\lambda}{4}\left[1 - \frac{1}{(1 + \frac{\mu^3\alpha_{in}}{3\sqrt{2}\beta\lambda}e^{\lambda y^-})} + \ln(1 + \frac{\mu^3\alpha_{in}}{3\sqrt{2}\beta\lambda}e^{\lambda y^-})\right]. \end{aligned} \tag{43}$$

For the limit, $y^- \to y_s^- - 0$, we obtain

$$H(y_s^- - 0) \approx M + \frac{\kappa\lambda}{4}(1 - e^{-\frac{4M}{\kappa\lambda}}) \tag{44}$$

which is greater than the total mass of the black hole. Therefore, the remaining mass of black hole at this point is negative. The integrated Hawking flux $H(y^-)$ is saturated after the black hole is completely evaporated and the total Hawking flux is a just $H(y_s^-)$. We approximately evaluated the integrated Hawking flux since we did not obtain the exact intersection point.

It is convenient to write the Bondi energy as [14]

$$B(y^-) = \sqrt{\kappa}(\lambda + \partial_- - \partial_+)\delta\Omega(y^+, y^-)|_{y^+ \to +\infty} \tag{45}$$

where $\delta\Omega(y^+, y^-) = \Omega(y^+, y^-) - \bar{\Omega}(y^+, y^-)$. Note that the contribution to the Bondi energy from the past null infinity is zero since for $x^+ \to -\infty$, the geometry is LDV as we see from (29). By putting Eq. (30) into (45) in the asymptotically static coordinates (40), we obtain Bondi energy,

$$B(y^-) = M - \frac{\kappa\lambda}{4}\left[\frac{\frac{\mu^3\alpha_{in}}{3\sqrt{2}\beta\lambda}}{(\frac{\mu^3\alpha_{in}}{3\sqrt{2}\beta\lambda} + e^{-\lambda y^-})} + \ln(1 + \frac{\mu^3\alpha_{in}}{3\sqrt{2}\beta\lambda}e^{\lambda y^-})\right]. \tag{46}$$

Note that at the point $y_s^- - 0$, the Bondi energy is approximately given by

$$B(y_s^- - 0) \approx -\frac{\kappa\lambda}{4}(1 - e^{-\frac{4M}{\kappa\lambda}}) \tag{47}$$

which is negative [15]. For $y^- > y_s^-$, the Bondi energy is zero since the negative energy of black hole is emitted through the thunderpop (39) and the final state of black hole becomes the shifted LDV. On the other hand, from Eqs. (43) and (46), we see that the sum of integrated Hawking radiation and Bondi energy is conserved, and independent of time. Naturally we see that the total energy (ADM) [17] is conserved through the evaporation of the black hole.

In our black hole system, there are two kinds of outgoing radiations, classical thunderpop and quantum-mechanical Hawking radiation. Further, we see that the carrier of classically infalling energy is the infalling soliton and may naturally expect that the outgoing classical thunderpop is an outgoing soliton. Let us now check briefly whether or not the thunderpop energy can be the soliton energy. The outgoing soliton of negative energy is given by

$$T^f_{--} \equiv -\frac{1}{2}(\partial_- f)^2 = -\frac{\mu^4}{16\beta\alpha_{out}^2} sech^4(\Delta_{out} - \Delta_s) \tag{48}$$

where $\Delta_s = \Delta(x_s^+, x_s^-)$. The energy density falls off zero at the asymptotically null infinity. In the shock wave limit, however, it reaches when v_{out} approaches the velocity of light where the soliton energy density is given by

$$T^f_{--}(x^+, x^-) \to -\frac{\mu^3}{3\sqrt{2}\beta\alpha_{out}} \delta(x^- - x_s^-). \tag{49}$$

In terms of the asymptotically static coordinates (40), the soliton energy is given by

$$\int dy^- T^f_{--}(y^+, y^-) = -\frac{\mu^3}{3\sqrt{2}\beta\alpha_{out}} e^{-\lambda y_s^-}. \tag{50}$$

In our model, the soliton energy can be identified with the thunderpop energy if the following condition is met,

$$-\frac{\mu^3}{3\sqrt{2}\beta\alpha_{out}} e^{-\lambda y_s^-} = -\frac{\kappa\lambda}{4}(1 - e^{-\frac{4M}{\kappa\lambda}}) \tag{51}$$

and it reduces to

$$cosh(\frac{4M}{\kappa\lambda}) = 1 + \frac{\mu^6}{9\beta^2\lambda^2\kappa}\left(\frac{\alpha_{in}}{\alpha_{out}}\right), \tag{52}$$

where we used the relation, $e^{\lambda y_s^-} = \frac{3\sqrt{2}\beta\lambda}{\mu^3\alpha_{in}}(e^{\frac{4M}{\kappa\lambda}} - 1)$. Because $\frac{\alpha_{in}}{\alpha_{out}} \gg 1$, the black hole mass is

$$M \approx \frac{\kappa\lambda}{4} \ln(2 + \frac{2\mu^6}{9\beta^2\lambda^2\kappa})\left(\frac{\alpha_{in}}{\alpha_{out}}\right) \tag{53}$$

which satisfies the relation (36). So we may think that the thunderpop can be a classical outgoing soliton while the Hawking radiation is a quantum mechanical one.

The finial point to be mentioned is about the conserved topological charge which characterizes the soliton. We apply the method developed in Ref. [16] to the topological current in order to obtain the conservation relation. The topological soliton charge is defined by

$$\begin{aligned} Q_{Top}(t) &= \int_{-\infty}^{+\infty} dy^1 J^0(t, y^1) \\ &= \sqrt{\frac{\beta}{4\mu^2}} \left[f(t,\infty) - f(t,-\infty) \right] \end{aligned} \tag{54}$$

where the topological current is defined by $J^\mu = \sqrt{\frac{\beta}{4\mu^2}} \epsilon^{\mu\nu} \partial_\nu f$ and we assume that the quantity is defined in the asymptotically static coordinate for convenience. In the future null infinity, we define the following Bondi charge which corresponds to the Bondi energy,

$$\begin{aligned} Q_B(y^-) &= \frac{1}{2} \int_{-\infty}^{+\infty} dy^+ J^-(y^+, y^-) \\ &= \sqrt{\frac{\beta}{4\mu^2}} \left[f(\infty, y^-) - f(-\infty, y^-) \right], \end{aligned} \tag{55}$$

by integrating the current along the null line. Then we define the radiation charge which corresponds to the integrated Hawking radiation,

$$\begin{aligned} Q_R(y^-) &= \frac{1}{2} \int_{-\infty}^{y^-} dy^- J^+(\infty, y^-) \\ &= \sqrt{\frac{\beta}{4\mu^2}} \left[f(\infty, -\infty) - f(\infty, y^-) \right]. \end{aligned} \tag{56}$$

By using the eqs (54)-(56), the following conservation relation as an identity holds,

$$\begin{aligned} Q_{Top}(t) &= Q_B(y^-) + Q_R(y^-) + \sqrt{\frac{\beta}{4\mu^2}} \left[f(-\infty, y^-) - f(-\infty, \infty) \right] \\ &= Q_B(y^-) + Q_R(y^-), \end{aligned} \tag{57}$$

where we used the fact that our soliton solution (13) satisfies the condition, $f(-\infty, y^-) = f(-\infty, \infty)$.

In our model, the soliton charge is 1, which is conserved if the classical thunderpop is identified with the soliton. For $y^- < y_s^-$, $Q_B(y^-) = 1$ and $Q_R = 0$ since there is no gradient for the outgoing soliton solution with a speed of light. For $y^- > y_s^-$, $Q_B(y^-) =$

0 and $Q_R = 1$ due to the gradient of outgoing soliton solution. So the conservation relation of the topological charge naturally holds with the help of thunderpop.

We studied the back reaction of the metric due to the Hawking radiation in the black hole induced by an infalling topological soliton. The Hawking radiation and Bondi energy were calculated along the null lines and the sum of them is conserved. The finial state of the black hole is not different from that of the RST model. However, the outgoing (classical) thunderpop may carry an soliton charge. This situation is natural since the source of classical thunderpop is given only by the classical soliton in our model whereas the quantum mechanical radiation may come from the Hawking radiation which does not carry the topological charge. So the infalling topological charge may be emitted by the outgoing thunderpop.

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References

[1] C. G. Callan, S. B. Giddings, J. A. Harvey, and A. Strominger, Phys. Rev. **D 45** (1992) R1005.

[2] S. W. Hawking, Commun. Math. Phys. **43** (1975) 199.

[3] J. G. Russo, L. Susskind, and L. Thorlacius, Phys. Rev. **D 46**, 3444 (1992); Phys. Rev. **D 47** (1993) 533.

[4] H.-S. Shin and K.-S. Soh, "Black hole formation by Sine-Gordon solitons in two-dimensional dilaton gravity, hep-th/9501045 (to appear Phys. Rev. D); H.-M. Johng, H.-S. Shin, and S.-K. Soh, "Black holes in two-dimensional dilaton gravity and nonlinear Klein-Gordon soliton", hep-th/9504109.

[5] C. Vaz and L. Witten, "Soliton induced singularities in 2d gravity and their evaporation", gr-qc/9504037.

[6] R. Arnowitt, S. Deser and C. W. Misner, " The Dynamics of General Relativity" in *Gravitation: An Introduction to Current Research*, ed. L. Witten, (John Wiley & Sons, New York, 1962).

[7] R. M. Wald, Ann. Phys. (N.Y.) 110 (1978) 472; Phys. Rev. **D 17** (1978) 1477.

[8] A. Strominger, Phys. Rev. **D 46** (1992) 4396.

[9] S. Liberati, Phys. Rev. **D 51** (1995) 1710.

[10] A. Bilal and C. Callan, Nucl. Phys. **B394** (1993) 73.

[11] S. P. de Alwis, Phys. Rev. **D 46** (1992) 5429.

[12] H. Bondi, M. G. Van der Burg and A. W. K. Metzner, Proc. Roy. Soc. Lond. **A 269** (1962) 21.

[13] S. W. Hawking, Phys. Rev. Lett. **69** (1992) 406.

[14] W. T. Kim and J. Lee, "Hawking radiation and energy conservation in an evaporating black hole", hep-th/9502115 (to appear Phys. Rev. D).

[15] S. P. de Alwis, Phys. Rev. **D 49** (1994) 941.

[16] W. T. Kim and J. Lee, "ADM, Bondi, and energy conservation in two-dimensional dilaton gravities, hep-th/9502078.

[17] E. Witten, Phys. Rev. **D 44** (1991) 314; R. B. Mann, Phys. Rev. **D 47** (1993) 4438; Y. Park and A. Strominger, Phys. Rev. **D 47** (1993) 1569; D. Bak, D. Cangemi, and R. Jackiw, Phys. Rev. **D 49** (1994) 5173; D. Louis-Martinez and G. Kunstatter, "Two dimensional dilaton gravity coupled to an abelian gauge field", gr-qc/9503016.